Optics: Approximating the Focal Length of a Thin Lens as a Function of Wavelength

The full equation for the focal length of a thick lens is:\(^1,2\)

\[
\frac{1}{f} = (n-1)\left(\frac{1}{r_1} - \frac{1}{r_2}\right) + \frac{d(n-1)^2}{nr_1r_2} \quad (1)
\]

Here \(f\) is the focal length, \(n\) is the index of refraction, \(r_1\) and \(r_2\) are the radii of the first and second surfaces and \(d\) is the thickness of the lens.

In many cases, especially when the focal length is long, we can ignore the second term on the right-hand side of the equation and use only the first term as a thin lens approximation. When the focal length is long the radii of the two surfaces are much larger than the thickness \(d\) and the second term becomes negligible. In those cases, we can arrive at some simple equations for Newport catalog lenses.

Bi-convex:

\[
f = \frac{1}{n-1}\left(\frac{r}{2}\right)
\]

Plano-convex:

\[
f = \frac{r}{n-1}
\]

Bi-concave:

\[
f = \frac{1}{n-1}\left(\frac{-r}{2}\right)
\]

Plano-concave:

\[
f = \frac{-r}{n-1}
\]
Newport’s bi-convex and bi-concave lenses are symmetrical, so \( r_1 = r_2 \) in each case, simplifying those expressions. For the plano-convex and plano-concave lenses, \( r_2 \) is infinite. In the cases of the concave lens types, we use the minus sign to include explicitly the negative radii of curvature.

Each of these equations is proportional to \( 1/(n-1) \), the inverse of the index of refraction minus one. This allows an easy method to approximate the focal length as a function of wavelength. We can use that index proportionality in this equation:

\[
\frac{f(\lambda_1)}{f(\lambda_2)} = \frac{n(\lambda_2) - 1}{n(\lambda_1) - 1}
\]

The focal length of Newport BK7 and fused silica lenses is specified at 589 nm. We can then look up the index of refraction at 589 nm and at any other wavelength and use those two indices to approximate the focal length at that other wavelength. Tables with the indices of refraction of several optical materials can be found on the Newport website here:


As an example, let’s approximate the focal length of a BK7 lens at 1064 nm. From the tables found in the reference Technical Note, the index of refraction of BK7 at 589 nm is 1.51673, while the index at 1064 nm is 1.50663. Plugging these numbers in the above equation, we find:

\[
\frac{f(1064)}{f(589)} = \frac{0.51673}{0.50663} = 1.0199
\]

From this, we find that the focal length of a BK7 lens increases by just about 2% between 589 and 1064 nm. An increase in focal length, as seen here, is what we expect, since we know from Snell’s Law that when the index of refraction is less, there is also less bending of the light toward the optical axis, resulting in a longer focal length. As noted above, this approximation is strictly true when the thickness of the lens can be ignored compared to the radii of curvature, but it can still be a useful approximation even as you tend toward shorter focal lengths.

We conclude with a table with a few additional examples for lenses made with BK7 and UV fused silica (UVFS). The table includes the index for each material at the design wavelength of 589 nm and a few selected examples of second wavelength, with index at that wavelength and the approximate percentage change in focal length. You may find it instructive to verify the results in the last column.

<table>
<thead>
<tr>
<th>Material</th>
<th>n(589 nm)</th>
<th>( \lambda_2 )</th>
<th>n(( \lambda_2 ))</th>
<th>( \Delta f ) (approx)</th>
</tr>
</thead>
<tbody>
<tr>
<td>BK7</td>
<td>1.51673</td>
<td>850 nm</td>
<td>1.50984</td>
<td>+1.4%</td>
</tr>
<tr>
<td>BK7</td>
<td>1.51673</td>
<td>1300 nm</td>
<td>1.50346</td>
<td>+2.6%</td>
</tr>
<tr>
<td>UVFS</td>
<td>1.45840</td>
<td>355 nm</td>
<td>1.47612</td>
<td>-3.7%</td>
</tr>
<tr>
<td>UVFS</td>
<td>1.45840</td>
<td>850 nm</td>
<td>1.45250</td>
<td>+1.3%</td>
</tr>
<tr>
<td>UVFS</td>
<td>1.45840</td>
<td>1064 nm</td>
<td>1.44963</td>
<td>+2.0%</td>
</tr>
</tbody>
</table>

**Notes:**
See, for example:

2. Hecht, *Optics* (Addison Wesley), Chapter 6, More on Geometrical Optics